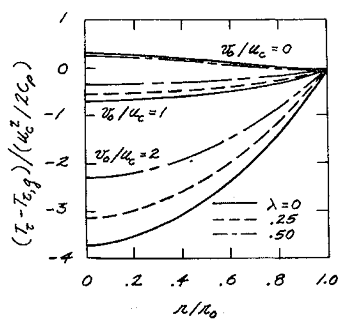

Fig. 1 Total temperature distribution for laminar slip flow through a rotating circular tube (Pr = 0.7)

creasing rotation;[4] however, with rarefaction or velocity jump ($\lambda \neq 0$), a diminution of the total temperature variation below the continuum flow value is experienced, and this decrease is accentuated with increasing slip. This effect of velocity jump on total temperature difference was observed earlier.[5]

The foregoing analysis suggests that a vortex tube using a slightly rarefied gas in which laminar slip flow occurs will exhibit a smaller energy separation effect than observed under laminar continuum flow conditions. A more detailed analytical study as well as an experimental study of vortex flows of a rarefied gas should prove illuminating.

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More on the Effectiveness Concept in Mass-Transfer Cooling

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Nomenclature

B_h = blowing rate for energy transfer $\equiv [(\rho v)_w/(\rho u)_\infty] \times (1/C_{h0})$
B_s = blowing rate based on sublayer thickness [cf., Eq. (7)]
$C_f/2$ = $\frac{1}{2} \times$ friction coefficient $\equiv \tau_w/\rho_\infty u_\infty^2$
C_h = Stanton number $\equiv [k_w(\partial T/\partial y)_w/(\rho u c_p)_\infty (T_r - T_w)]$
c_p = specific heat at constant pressure
k = thermal conductivity
Pr = Prandtl number $\equiv c_p\mu/k$
r = recovery factor $\equiv [2c_{p\infty}(T_r - T_\infty)/u_\infty^2]$
R = effectiveness $\equiv (T_w - T_c)/(T_{r0} - T_c)$
Re = Reynolds number $\equiv \rho u_\infty x/\mu$
T = temperature
T_c = temperature of coolant [cf., Eq. (1)]
T_r = temperature of wall for case in which temperature gradient vanishes at wall
u = velocity parallel to wall
v = velocity normal to wall
x = coordinate parallel to wall
y = coordinate normal to wall
μ = dynamic viscosity coefficient
ρ = density
τ = shearing stress

Superscripts

c = mixture component(s) (e.g., coolant) added at wall
$*$ = variable evaluated at reference state

Subscripts

s = sublayer boundary
w = wall
0 = limiting value as blowing rate approaches zero
∞ = outer edge of boundary layer

THE effectiveness R was used recently by Bartle and Leadon in heat transfer correlations for nitrogen[1] and foreign gas[2] injections into turbulent air streams. They report[2] that "the effectiveness is found to represent the data well for a wide variety of coolant gases, Mach numbers 2 and 3.2, and small Reynolds number variations, when it is considered to a function only of $B_h c_p^c/c_{p\infty}$." (Nomenclature of the present paper is used.) Tewfik[3] points out some limitations of this parameter and concludes that "it is not of much use in correlating the reduction in heat transfer with injection, except perhaps when the wall temperature is much different from adiabatic, or in the special case of air injection in low-speed flow." The purpose of the present note is to examine analytically the dependence of the effectiveness upon temperatures, blowing rate, Mach number, and Reynolds number.

The heat-balance equation

$$C_h^*\rho^* u_\infty c_p^* \,(T_r - T_w) = (\rho v c_p^c)_w \,(T_w - T_c) \tag{1}$$

may be rearranged[4] to obtain

$$R = \left[1 + \frac{(\rho v c_p^c)_w}{\rho^* u_\infty c_p^*} \frac{1}{C_h^*} \frac{T_{r0} - T_w}{T_r - T_w}\right]^{-1} \tag{2}$$

For turbulent Prandtl number equal to unity, Knuth and Dershin[4] write, as an extension of the Reynolds analogy to the case of mass addition at the wall,

$$1 + \frac{(\rho v c_p^c)_w}{\rho^* u_\infty c_p^*} \frac{1}{C_h^*} = \left[\left(1 + \frac{(\rho v)_w}{\rho^* u_\infty} \frac{2}{C_f^*} \frac{u_s}{u_\infty}\right)^{Pr^{*-1}} \times \left(1 + \frac{(\rho v)_w}{\rho^* u_\infty} \frac{2}{C_f^*}\right)\right]^{c_p^c/c_p^*} \tag{3}$$

Substituting from Eq. (3) into Eq. (2),

$$R = \left\langle 1 + \left\{\left[\left(1 + \frac{(\rho v)_w}{\rho^* u_\infty} \frac{2}{C_f^*} \frac{u_s}{u_\infty}\right)^{Pr^{*-1}} \times \left(1 + \frac{(\rho v)_w}{\rho^* u_\infty} \frac{2}{C_f^*}\right)\right]^{c_p^c/c_p^*} - 1\right\} \frac{T_{r0} - T_w}{T_r - T_w}\right\rangle^{-1} \tag{4}$$

This analytical result is to be compared with the empirical result

$$R = [1 + \tfrac{1}{3} B_h \,(c_p^c/c_{p\infty})]^{-3} \tag{5}$$

presented by Bartle and Leadon.[2] One might predict, using Eq. (4), values of R for the test conditions of Ref. 1 and compare these predicted values with the measured values. (Attempts to predict for the test conditions of Ref. 2 are held in abeyance until a reference-composition expression for turbulent flows is established.) Such predictions are more valuable if they can be made knowing only the design conditions

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(including blowing rate) and zero-blowing values of transport parameters (including friction coefficient and recovery factor). Hence, values of C_f^*/C_{f0}^* and r^*/r_0^*, required in evaluating Eq. (4), are to be predicted also. It has been shown[4] that, at least for small to modest blowing rates,

$$1 + \frac{(\rho v)_w}{\rho^* u_\infty} \frac{2}{C_f^*} \frac{u_s}{u_\infty} \approx \frac{C_{f0}^*}{C_f^*} \approx \exp(B_s^*) \qquad (6)$$

with

$$B_s^* \equiv 11.5 \frac{(\rho v)_w}{\rho^* u_\infty} \left(\frac{2}{C_{f0}^*} \frac{2}{C_f^*}\right)^{1/4} \qquad (7)$$

(Although Fig. 8 of Ref. 4 is considered to be the best available correlation of friction coefficient with blowing rates, it cannot be used in the present calculation; most of the data used in preparing Fig. 8 are the same data with which results of the present calculation are to be compared.) In the absence of established empirical relations, the authors use[4]

$$r^*/r_0^* \approx 1 - \tfrac{1}{3}(1 - Pr^*)B_s^* \qquad (8)$$

in the required prediction of r^*/r_0^*. [This equation represents, to good approximation, the equation obtained by setting, in Eq. (21) of Ref. 4, the turbulent Prandtl number equal to unity and $c_p^c/c_p^* = 1$, expanding in series, and retaining only first-order terms in blowing rates.] Substituting from Eqs. (6) and (8) into Eq. (4) and setting $c_p^c/c_p^* = 1$, one obtains

$$R = \left\{1 + \left[\exp[(Pr^* - 1)B_s^*] \times \left(1 + \frac{(\rho v)_w}{\rho^* u_\infty} \frac{2}{C_{f0}^*} \exp(B_s^*)\right) - 1\right] \times \left[1 - \frac{1}{3}(1 - Pr^*) \frac{T_{r0} - T_\infty}{T_{r0} - T_w} B_s^*\right]^{-1}\right\}^{-1} \qquad (9)$$

Equation (9) relates, for $c_p^c/c_p^* = 1$, the effectiveness to design conditions and zero-blowing values of transport parameters.

Values of R were computed using values of $(\rho v)_w/(\rho u)_\infty$ and C_{f0} measured by Bartle and Leadon[1] and evaluating fluid properties at the reference temperature predicted by the expression developed by Knuth[5] which becomes, for $c_p^c/c_p^* = 1$,

$$T^* = 0.5(T_w + T_\infty) + 0.2(T_{r0} - T_\infty) + 0.1B_h^*(T_w - T_\infty) \qquad (10)$$

These predicted values are compared with the empirical curve of Bartle and Leadon in Fig. 1. Deviations at high blowing rates are due mainly to the inaccuracy of Eq. (6) at high blowing rates. The accuracy of future predictions would be improved by replacing Eq. (6) with

$$C_f^*/C_{f0}^* = (1 + \tfrac{1}{3}B_s^*)^{-3} \qquad (11)$$

suggested by Fig. 8 of Ref. 4.

These results suggest, for the test conditions of Ref. 1, an approximate equivalence of Eq. (4), derived analytically in Ref. 4, and Eq. (5), derived empirically in Ref. 1. An examination of Eqs. (9) and (10) indicates, however, that the following three limitations might apply:

1) As emphasized by Tewfik,[3] for T_c near T_{r0}, the value of R depends strongly upon the value of $(T_{r0} - T_w)/(T_r - T_w)$. Equation (10) indicates that a sufficient condition for weak dependence of R on these temperatures is given by

$$\left|\frac{1}{3}(1 - Pr^*) \frac{T_{r0} - T_\infty}{T_{r0} - T_w} B_s^*\right| \ll 1 \qquad (12)$$

2) The apparent Mach number independence of the blowing parameter B_h observed by Bartle and Leadon can be explained if one notes that, to the extent that variations of heat capacities and Prandtl numbers with temperature may be neglected,

$$B_h = B_h^*(T_0^*/T^*)^{0.7} \qquad (13)$$

But, from Eq. (10),

$$\frac{T^*}{T_0^*} = 1 + \frac{T_w - T_\infty}{5(T_w + T_\infty) + 2(T_{r0} - T_\infty)} B_h^* \qquad (14)$$

For the test conditions of Ref. 1, this ratio is relatively insensitive to the value of the Mach number. If T^*/T_0^* differs appreciably from unity, then fluid properties must be evaluated at the reference temperature T^*.

3) The apparent Reynolds number independence of the correlation presented by Bartle and Leadon can be explained also if one notes that

$$\frac{C_{f0}^*}{2} = \frac{C_{f0}}{2} \left(\frac{T_0^*}{T^*}\right)^{0.7} \frac{T^*}{T_\infty} \qquad (15)$$

For the test conditions of Ref. 1, C_{f0}^* varied by only about 20%. In general, and as indicated by Bartle and Leadon in Fig. 10 of Ref. 2, if C_{f0}^* varies significantly in a series of tests, then this variation will have to be considered in a correlation of the heat transfer data.

In conclusion, for the test conditions of Ref. 1, the empirical curve for effectiveness proposed by Bartle and Leadon compares favorably with predictions of the expression developed analytically by Knuth and Dershin. The theoretical expression suggests, however, that several limitations may have to be placed on the range of test conditions for which the empirical equation (5) is applicable. Since the validity of the theoretical equation (4) has been proven only for the test conditions of Ref. 4, it remains for the experimental investigators to settle these questions.

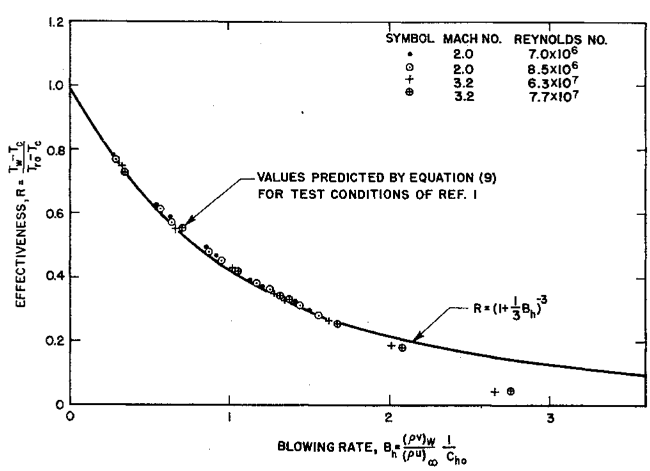

Fig. 1 Values of effectiveness R as function of blowing-rate parameter B_h as predicted by modified Reynolds analogy for test conditions of Ref. 1

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Surface Mass-Transfer Correlations

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Nomenclature

C_p = specific heat at constant pressure
F = mass-injection ratio, $(\rho v)_c/(\rho u)_1$
G = nondimensional injection variable, $C_{p_c}F/C_{p_1}St_0$
m = molecular weight
n = number of molecules per unit volume
q = surface heat flux
St = Stanton number
T = temperature
u = velocity along the surface
v = velocity normal to the surface
Z = corrected molar injection rate, $0.6\ m_1C_{p_1}G/m_cC_{p_c}$ or $0.6\ n_cv/n_1u_1St_0$
ρ = density

Subscripts

a = adiabatic
c = coolant
0 = zero injection condition
w = surface value
1 = freestream value

Bartle and Leadon[1] have reported experimental turbulent mass-transfer cooling data obtained at a Mach number of 3.2 using nine different coolants. Having found that no extant turbulent boundary layer analysis adequately predicts the heat transfer reductions measured and that the use of the cooling effectiveness instead of the Stanton number reduction, St/St_0, circumvents the "cranky nature" of the latter and "minimizes the effect of Reynolds number," they recommend that the engineer make direct use of their empirical effectiveness formula. The local heat-flow reduction, q/q_0, then is to be obtained readily from Eq. (1):

$$\text{effectiveness} = \frac{T_w - T_c}{T_{aw0} - T_c} = \left(1 + \frac{C_{p_c}F}{C_{p_1}St_0}\,\frac{q_0}{q}\right)^{-1} \quad (1)$$

Tewfik[2] criticizes the conclusions of Ref. 1, pointing out that the effectiveness depends strongly on the wall-temperature level near adiabatic conditions. Although Tewfik's comments are not incorrect, they miss the mark. The engineer, to whom the recommendations of Bartle and Leadon are addressed, is interested in injection cooling for the very purpose of making the wall temperature much different from the adiabatic wall temperature. Effectively, Tewfik finds no serious fault with the recommendations in this regime.

It should be pointed out that there is, in fact, a basic objection to some of the conclusions of Ref. 1. It is suggested therein that the experimental data correlate much better when presented as effectiveness rather than as heat-flow reduction. It is clear from Eq. (1), however, that when the effectiveness is small as compared with unity (as it is for much of the experimental measurements) the percentage scatter in the two quantities is inherently almost the same.

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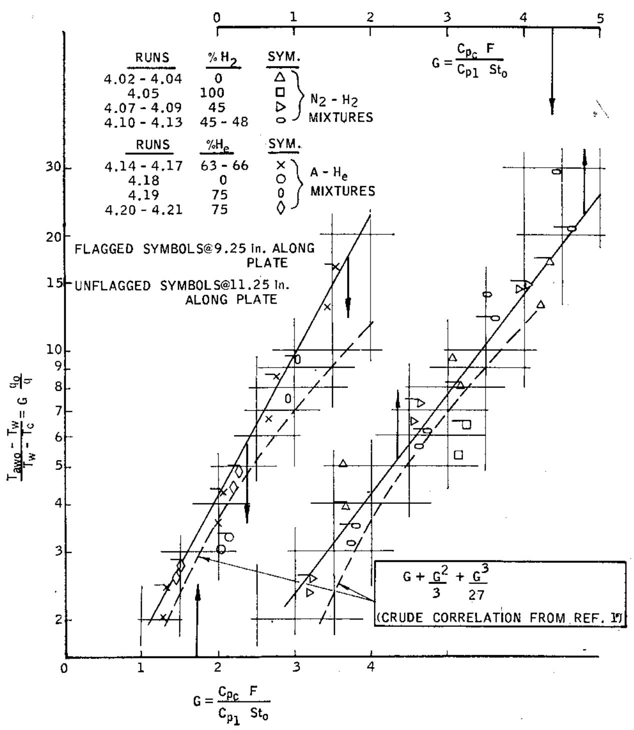

Fig. 1 Corrected heat-flow reduction for monatomic and diatomic gases

What may have been in mind in presenting a single correlating curve for the effectiveness of all nine coolants is that it need not be known too precisely for some engineering purposes when it is small. Be that as it may, it is quite misleading to suggest that the heat-flux reduction itself is evaluated properly by using such a crude correlation.

Take a closer look at the portion of the heat-flow reduction data covering monatomic and diatomic gas injection (66% of the total). The monatomic gas injection data are plotted at the left in Fig. 1 and the diatomic, at the right. The latter has two salient features: 1) the data scatter greater at 11.25 in. along the plate than at 9.25 in., and 2) the lack of a significant trend as the composition of the injected gas changes from pure nitrogen to pure hydrogen. As can be seen, giving special emphasis to the data at the earlier location allows a single straight correlating line to be drawn. The corresponding correlation for monatomic gas injection is shown at the left. By way of contrast, the dashed lines present the crude correlation of Bartle and Leadon.

In the context of the basic study of Ref. 3, the single correlation for all the diatomic gas injection data indicates that, in this operating regime, i.e., high molar injection rates into turbulent flow, injection fluids achieve the same corrected heat-flow reduction q/Gq_0 when their molar injection rates are the same even though the fluids themselves are not similar thermophysically. Consistent with this finding, the slope of the correlating line for monatomic gas injection is exactly 1.4 times that for diatomic gas injection. These results suggest that the volumetric displacement mechanism of mass transfer for achieving surface effects equivalent to the slip and temperature-jump characteristics of low-density fluid mechanics is more effective in turbulent flow than in laminar flow.

The data correlations for monatomic and diatomic gas injection may be unified into a single correlation of the heat-flow reduction as a function of the corrected molar injection rate Z as Eq. (2):

$$q/q_0 = 1.3(m_1\,C_{p_1}/m_c\,C_{p_c})^{1/2}Ze^{-Z} \qquad (Z > 1) \quad (2)$$

In view of the scatter of the data, however, there is some ques-